Recursive induction:

ex)

$f(0) = 2$
$f(1) = 3$ } generally base cases align

$\forall n \geq 2,\ \boxed{f(n) = 3f(n-1) - 2f(n-2)}$

in book: $f(n+1) = 3f(n) - 2f(n-1)$

Claim: $\forall n \in \mathbb{N}\ f(n) = 2^n + 1$ closed form

Proof by induction on n.

Base cases: $f(n) = 2^n + 1$ when $n=0$ and $n=1$.

$f(0) = 2^0 + 1 = 2 = f(0)$

$f(1) = 2^1 + 1 = 3 = f(1)$

IH: Suppose closed form of $f(n)$ is 2^n+1 for $n = 1 \dots k-1$.

Goal: we want to show $f(k) = 2^k+1$, or

$f(k) = 3f(k-1) - 2f(k-2) = 2^k + 1$

$f(k) = 3\underbrace{f(k-1)}_{IH} - 2\underbrace{f(k-2)}_{IH}$

Applying the IH twice, we get $f(k) = 3(2^{k-1}+1) - 2(2^{k-2}+1)$

$f(k) = 3 \cdot 2^{k-1} + 3 - 2 \cdot 2^{k-2} - 2$

$3 \cdot 2^{k-1} + 1 + (-1)(2)(2^{k-2})$

$3 \cdot 2^{k-1} + 1 - 2^{k-1}$

$2 \cdot 2^{k-1} + 1 = 2^k + 1$